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Supersymmetric coupling of a self-dual string to a $(2,0)$ tensor multiplet background

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Abstract: We construct an interaction between a $(2,0)$ tensor multiplet in six dimensions and a self-dual string. The interaction is a sum of a Nambu-Goto term, with the tension of the string given by the modulus of the scalar fields of the tensor multiplet, and a non-local Wess-Zumino term, that encodes the electromagnetic coupling of the string to the two-form gauge field of the tensor multiplet. The interaction is invariant under global $(2,0)$ supersymmetry, modulo the equations of motion of a free tensor multiplet. It is also invariant under a local fermionic κ-symmetry, as required by the BPS-property of the string.

Keywords: Field Theories in Higher Dimensions, M-Theory, Extended Supersymmetry.

Contents

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1. Introduction

The six-dimensional $(2,0)$ theories are one of the most mysterious discoveries in string theory during the past decade, and finding the right framework to define them remains an outstanding challenge [1]. In previous publications [2, 3], we have pursued an approach to study these theories by working above a generic vacuum state, i.e. not at the origin of the moduli space, where they can be thought of as describing the dynamics of massless particles and tensile strings. The existence of these constituents follows directly from the $(2,0)$ supersymmetry algebra [4]. At energies low compared to the scale set by the string tension, the theory is free and well understood, but at higher energies the interactions between the particles and the strings must be taken into account. In the present paper, we will construct the leading such interactions.

To be more specific, we will focus our attention on the simplest A_1 version of $(2,0)$ theory. The particles then arise upon quantization of a single $(2,0)$ tensor multiplet, i.e. a set of scalar fields ϕ in the **5** representation of the SO(5) R-symmetry group, a set of chiral spinors ψ in the **4** representation of SO(5), and a two-form gauge field b in the singlet **1** representation of SO(5). The field strength $h = db$ of b can be decomposed into its self-dual part $h_+ = \frac{1}{2}(h + *h)$ and anti self-dual part $h_- = \frac{1}{2}(h - *h)$. The anti self-dual part h_- is actually not part of the tensor multiplet, but it is convenient to include it nevertheless as a decoupled 'spectator' field. One can then construct a supersymmetric free action for these fields [3]:

$$S_{\mathrm{TM}} = \int_M (d\phi \cdot \wedge * \, d\phi + h \wedge *h + \psi \not{D} \psi) \, , \tag{1.1}$$

where the integral is over six-dimensional Minkowski space M and the bilinears in the fields are SO(5) invariants. The coefficients of the kinetic terms for ϕ and ψ can be absorbed by changing the normalization of these fields (which take their values in linear spaces) and are thus of no consequence. However, b is not really a tensor field but rather a 'connection', so its normalization has an absolute meaning. The coefficient of its kinetic term is a priori a parameter of the theory, but it turns out that its value is uniquely determined by the consistent decoupling of h_- [5].

The A_1 version of $(2,0)$ theory also contains a single type of string. The degrees of freedom of such a string is an embedding map X from the string world-sheet Σ to Minkowski space M and a set of world-sheet fermions Θ that transform as an anti-chiral spinor in Minkowski space and in the **4** of SO(5). Actually, two of the six components of X can be eliminated by fixing the reparametrization invariance of the world-sheet Σ. Similarly, half of the components of Θ can be eliminated by fixing a local fermionic κ-symmetry on Σ.

The couplings between the world-sheet degrees of freedom X and Θ and the Minkowski space fields ϕ, b, and ψ are dictated by the following three principles [2]:

- The tension of the string should be given by the local value of $\sqrt{\phi \cdot \phi}$.
- The string should couple electrically and magnetically to b in such a way that the anti self-dual part h_- of $h = db$ remains decoupled.
- The interactions should be supersymmetric.

In the present paper, we will not solve this problem completely. What we will do is to construct an interaction that is supersymmetric provided that the tensor multiplet fields fulfill the free equations of motion following from the action (1.1). But when this interaction is added to the action, the equations of motion are altered and the interaction is no longer supersymmetric. Nevertheless, our model can be trusted for computing scattering of tensor multiplet particles off a string to lowest order in their energy, and we intend to present such a calculation in a forthcoming publication. We also believe that the present work is a step towards constructing the exactly supersymmetric model, and hope to be able to return to this issue in the future.

To motivate our construction, it is convenient to first neglect the fermionic degrees of freedom, i.e. ψ and Θ, and concentrate on fulfilling the first two requirements. It is then straightforward to comply with the first requirement by constructing a kinetic term for the string of Nambu-Goto type:

$$S_{\rm NG} = \int_\Sigma d^2\sigma \sqrt{\phi \cdot \phi} \sqrt{-G}\,. \tag{1.2}$$

Here, the world-sheet Σ is parametrized by σ, and G is the determinant of the metric on Σ induced by its embedding into Minkowski space M. The coefficient of this term will eventually be determined by the requirement of κ-symmetry.

To incorporate the electromagnetic coupling, we start with the standard electric coupling of the string to the field b:

$$S_{\rm el} = \int_\Sigma b\,, \tag{1.3}$$

where a pull-back of the Minkowski space field b to the world-sheet Σ by the embedding map is understood. Let now D be an open three-manifold embedded in Minkowski space M such that its boundary is given by Σ, i.e. $\partial D = \Sigma$. We can then rewrite the electric coupling by using Stokes' theorem as

$$S_{\rm el} = \int_D h\,, \tag{1.4}$$

where a pull-back to D is understood. This expression for the electric coupling can be used also in a topologically non-trivial context, where the connection b cannot be represented by a globally defined two-form. One must then ensure that, given Σ, the choice of three-manifold D has no observable consequences. In the usual way, this means that the coefficient of the electric coupling is quantized to integer values. Given this form of the electric coupling, it is natural to introduce a corresponding magnetic coupling as

$$S_{\rm mag} = \int_D *h\,. \tag{1.5}$$

The three-manifold D can thus be interpreted as the world-volume of a 'Dirac membrane' ending on the string [6]. Altogether, we have an electromagnetic coupling of Wess-Zumino type:

$$S_{\rm WZ} = \int_D f\,, \tag{1.6}$$

where $f = (h + *h) = 2h_+$. This term thus manifestly leaves the anti self-dual part h_- of h decoupled as required. It should be noted that the three-form f is closed, $df = 0$, when the field strength h fulfills the Bianchi identity $dh = 0$ and the free equation of motion $d(*h) = 0$.

The supersymmetrization of the interaction terms S_{NG} and $S_{\rm WZ}$ is analogous to many similar situations where branes are coupled to background fields [7]–[9]. Following [10], we introduce $(2,0)$ superspace and a constrained superfield Φ with component fields ϕ, ψ and h_+. We also construct a closed super-three-form F out of Φ. The string degrees of freedom X and Θ are considered as describing the embedding of the string world-sheet Σ into superspace. Also, Σ is the boundary of a three-manifold D embedded into superspace. In this way, the interaction terms can still be written as the sum of a Nambu-Goto term (1.2) and a Wess-Zumino term (1.6), with ϕ and f replaced by the superfields Φ and F. Supersymmetry is then manifest for each of these terms separately. The linear combination $S_{NG} + S_{\rm WZ}$ is also invariant under a local fermionic κ-symmetry. The parameter of this symmetry has the same representation content as Θ, but a certain projection has to be imposed that eliminates half of the components. The remaining symmetry is thus precisely sufficient to put half of the components of Θ to zero, as required by supersymmetry.

2. The supergeometry

2.1 Superspace

Following [10], we introduce a superspace with bosonic coordinates $x^{\alpha\beta} = -x^{\beta\alpha}$ and fermionic coordinates θ^α_a, where $\alpha, \beta = 1, \ldots, 4$ are SO(5,1) chiral spinor indices and $a = 1, \ldots, 4$ is an SO(5) spinor index. The latter index is raised and lowered from the left

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using the antisymmetric SO(5) invariant tensor Ω^{ab} and its inverse Ω_{ab}, i.e.

$$\begin{aligned}\theta^{a\alpha} &= \Omega^{ab}\theta^{\alpha}_{b} \\ \theta^{\alpha}_{a} &= \Omega_{ab}\theta^{b\alpha}\,.\end{aligned} \tag{2.1}$$

For consistency, this requires that $\Omega_{ab}\Omega^{bc} = \delta_a{}^c$.

Supersymmetry transformations are generated by the supercharges

$$Q^a_\alpha = \partial^a_\alpha - i\Omega^{ab}\theta^\beta_b\partial_{\alpha\beta}\,, \tag{2.2}$$

where $\partial_{\alpha\beta} = -\partial_{\beta\alpha}$ denotes the ordinary bosonic derivative and ∂^a_α a derivative with respect to the fermionic coordinate θ^α_a. The supercharges obey the anticommutation relations

$$\left\{Q^a_\alpha, Q^b_\beta\right\} = -2i\Omega^{ab}\partial_{\alpha\beta}\,. \tag{2.3}$$

We also need the superspace covariant derivatives

$$D^a_\alpha = \partial^a_\alpha + i\Omega^{ab}\theta^\beta_b\partial_{\alpha\beta}\,, \tag{2.4}$$

obeying

$$\left\{D^a_\alpha, D^b_\beta\right\} = 2i\Omega^{ab}\partial_{\alpha\beta}\,. \tag{2.5}$$

2.2 Superfields

After these preliminaries, we introduce the superfield $\Phi^{ab} = -\Phi^{ba}$ which is a function of the superspace coordinates x and θ. This superfield is subject to the algebraic constraint

$$\Omega_{ab}\Phi^{ab} = 0\,, \tag{2.6}$$

and the differential constraint

$$D^a_\alpha\Phi^{bc} + \frac{1}{5}\Omega_{de}D^d_\alpha\left(2\Omega^{ab}\Phi^{ec} - 2\Omega^{ac}\Phi^{eb} + \Omega^{bc}\Phi^{ea}\right) = 0\,. \tag{2.7}$$

It is useful to introduce two additional superfields Ψ^a_α and $H_{\alpha\beta}$, related to Φ^{ab} by the relations

$$\begin{aligned}\Psi^c_\alpha &= -\frac{2i}{5}\Omega_{ab}D^a_\alpha\Phi^{bc} \\ H_{\alpha\beta} &= \frac{1}{4}\Omega_{ab}D^a_\alpha\Psi^b_\beta\,.\end{aligned} \tag{2.8}$$

It can be shown that $H_{\alpha\beta} = H_{\beta\alpha}$. It thus transforms as a self-dual three-form. (An anti self-dual three-form would be denoted by $H^{\alpha\beta}$.) The differential constraint (2.7) yields that

$$D^a_\alpha\Phi^{bc} = -i\left(\Omega^{ab}\Psi^c_\alpha + \Omega^{ca}\Psi^b_\alpha + \frac{1}{2}\Omega^{bc}\Psi^a_\alpha\right) \tag{2.9}$$

$$D^a_\alpha\Psi^b_\beta = 2\partial_{\alpha\beta}\Phi^{ab} - \Omega^{ab}H_{\alpha\beta} \tag{2.10}$$

$$D^a_\alpha H_{\beta\gamma} = i\left(\partial_{\alpha\beta}\Psi^a_\gamma + \partial_{\alpha\gamma}\Psi^a_\beta\right), \tag{2.11}$$

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along with the equations of motion

$$\partial_{[\alpha\beta}\partial_{\gamma\delta]}\Phi^{ab} = 0 \tag{2.12}$$

$$\partial_{[\alpha\beta}\Psi^{a}_{\gamma]} = 0 \tag{2.13}$$

$$\partial_{[\alpha\beta}H_{\gamma]\delta} = 0\,. \tag{2.14}$$

The fact that the equations of motion appear is a consequence of having on-shell supersymmetry (i.e. the supersymmetry algebra only closes on-shell).

Denoting the lowest components (i.e. those independent of θ) of Φ, Ψ and H by ϕ, ψ and h, respectively, we can obtain an explicit expression for Φ in terms of these:

$$\begin{aligned}\Phi^{ab} = {} & \phi^{ab} - i\theta^{\alpha}_{c}\left(\Omega^{ca}\psi^{b}_{\alpha} + \Omega^{bc}\psi^{a}_{\alpha} + \frac{1}{2}\Omega^{ab}\psi^{c}_{\alpha}\right) + \\ & + i\theta^{\alpha}_{c}\theta^{\beta}_{d}\left(h_{\alpha\beta}\left(\Omega^{da}\Omega^{bc} + \frac{1}{4}\Omega^{dc}\Omega^{ab}\right) - \Omega^{da}\partial_{\alpha\beta}\phi^{bc} - \Omega^{db}\partial_{\alpha\beta}\phi^{ca}\right) + \\ & + \frac{1}{6}\theta^{\alpha}_{c}\theta^{\beta}_{d}\theta^{\gamma}_{e}\left(\Omega^{ab}\Omega^{ce}\partial_{\alpha\beta}\psi^{d}_{\gamma} + \Omega^{ac}\left(\Omega^{bd}\partial_{\alpha\beta}\psi^{e}_{\gamma} + 2\Omega^{de}\partial_{\alpha\beta}\psi^{b}_{\gamma} + 4\Omega^{eb}\partial_{\alpha\beta}\psi^{d}_{\gamma}\right) - \right. \\ & \left. - \Omega^{bc}\left(\Omega^{ad}\partial_{\alpha\beta}\psi^{e}_{\gamma} + 2\Omega^{de}\partial_{\alpha\beta}\psi^{a}_{\gamma} + 4\Omega^{ea}\partial_{\alpha\beta}\psi^{d}_{\gamma}\right)\right) + \mathcal{O}\left(\theta^{4}\right), \end{aligned} \tag{2.15}$$

where all higher order terms can be expressed in terms of ϕ, ψ and h. A priori, the final term in this sum is of order θ^{16}.

A general superfield transforms according to

$$\delta\Phi^{ab} = \left[\eta^{\gamma}_{c}Q^{c}_{\gamma}, \Phi^{ab}\right] \tag{2.16}$$

under a supersymmetry variation with a constant fermionic parameter η. For the lowest components of Φ, Ψ and H we get that

$$\delta\phi^{ab} = i\left(\Omega^{ac}\eta^{\alpha}_{c}\psi^{b}_{\alpha} - \Omega^{bc}\eta^{\alpha}_{c}\psi^{a}_{\alpha} - \frac{1}{2}\eta^{\alpha}_{c}\psi^{c}_{\alpha}\Omega^{ab}\right) \tag{2.17}$$

$$\delta\psi^{a}_{\alpha} = \Omega^{ab}\eta^{\beta}_{b}h_{\alpha\beta} + 2\partial_{\alpha\beta}\phi^{ab}\eta^{\beta}_{b} \tag{2.18}$$

$$\delta h_{\alpha\beta} = -i\eta^{\gamma}_{a}\left(\partial_{\alpha\gamma}\psi^{a}_{\beta} + \partial_{\beta\gamma}\psi^{a}_{\alpha}\right), \tag{2.19}$$

which coincides with the supersymmetry transformations of the tensor multiplet fields obtained in [3].

The free action

$$S_{\text{TM}} = \int d^{6}x\left\{-\Omega_{ac}\Omega_{bd}\partial_{\alpha\beta}\phi^{ab}\partial^{\alpha\beta}\phi^{cd} + 2h_{\alpha\beta}h^{\alpha\beta} - 4i\Omega_{ab}\psi^{a}_{\alpha}\partial^{\alpha\beta}\psi^{b}_{\beta}\right\} \tag{2.20}$$

is manifestly SO(5) invariant, and one may check that it is also invariant under the above supersymmetry transformations. The equations of motion following from this action coincide with (the lowest components of) eqs. (2.12)–(2.14). For a more elaborate discussion on these issues, along with reality and SO(5)-properties of the fields, we refer to [3].

2.3 Superforms

The next step is to set the stage for superforms. Let z^M be a (curved) coordinate in superspace, where ^{M} may take values ${}^{[\mu\nu]}$ (bosonic) and ${}^{\mu}_{m}$ (fermionic). Explicitly, we have the differentials

$$\begin{aligned} dz^{[\mu\nu]} &= dx^{\mu\nu} \\ dz^{\mu}_{m} &= d\theta^{\mu}_{m} \,. \end{aligned} \tag{2.21}$$

We also introduce the tangent space differentials E^A related to dz^M by

$$E^A = dz^M E_M{}^A \qquad \Leftrightarrow \qquad dz^M = E^A E_A{}^M \,, \tag{2.22}$$

where $E_M{}^A$ is the vielbein and its inverse is denoted by $E_A{}^M$. The tangent space index ^{A} also takes bosonic (${}^{[\alpha\beta]}$) and fermionic (${}^{\alpha}_{a}$) values. (The coordinates $z^{\alpha\beta} = x^{\alpha\beta}$ and $z^{\alpha}_{a} = \theta^{\alpha}_{a}$ are the same as those introduced in section 2.1.) In our case, we can choose the tangent space differential superforms as

$$\begin{aligned} E^{[\alpha\beta]} &= dx^{\alpha\beta} + i\Omega^{ab}\theta^{[\alpha}_{a} d\theta^{\beta]}_{b} \\ E^{\alpha}_{a} &= d\theta^{\alpha}_{a} \,, \end{aligned} \tag{2.23}$$

meaning that the vielbein $E_M{}^A$ has the components

$$\begin{aligned} E_{[\mu\nu]}{}^{[\alpha\beta]} &= \delta_{\mu}{}^{[\alpha}\delta_{\nu}{}^{\beta]} \\ E_{[\mu\nu]a}{}^{\alpha} &= 0 \\ E^{m[\alpha\beta]}_{\mu} &= -i\delta_{\mu}{}^{[\alpha}\delta_{\nu}{}^{\beta]}\Omega^{mn}\theta^{\nu}_{n} \\ E^{m\alpha}_{\mu a} &= \delta_{\mu}{}^{\alpha}\delta^{m}{}_{a} \,, \end{aligned} \tag{2.24}$$

while the inverse vielbein $E_A{}^M$ is

$$\begin{aligned} E_{[\alpha\beta]}{}^{[\mu\nu]} &= \delta_{\alpha}{}^{[\mu}\delta_{\beta}{}^{\nu]} \\ E_{[\alpha\beta]m}{}^{\mu} &= 0 \\ E^{a[\mu\nu]}_{\alpha} &= i\delta_{\alpha}{}^{[\mu}\delta_{\beta}{}^{\nu]}\Omega^{ab}\theta^{\beta}_{b} \\ E^{a\mu}_{\alpha m} &= \delta_{\alpha}{}^{\mu}\delta^{a}{}_{m} \,. \end{aligned} \tag{2.25}$$

The latter set of equations yields that the derivatives in tangent space coordinates,

$$D_A \equiv E_A{}^M \partial_M \,, \tag{2.26}$$

become

$$\begin{aligned} D_{[\alpha\beta]} &= \partial_{\alpha\beta} \\ D^{a}_{\alpha} &= \partial^{a}_{\alpha} + i\Omega^{ab}\theta^{\beta}_{b}\partial_{\alpha\beta} \,. \end{aligned} \tag{2.27}$$

The fermionic part agrees with the superderivative in eq. (2.4).

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A super-n-form is now written as

$$\omega = \frac{1}{n!} dz^{M_n} \wedge \cdots \wedge dz^{M_1} \omega_{M_1 \ldots M_n}, \tag{2.28}$$

where the wedge product is such that $dz^M \wedge dz^N = dz^N \wedge dz^M$ if both M and N are fermionic, otherwise $dz^M \wedge dz^N = -dz^N \wedge dz^M$. The exterior derivative d acts on the superforms according to

$$d\omega = \frac{1}{n!} dz^{M_n} \wedge \cdots \wedge dz^{M_1} \wedge dz^N \partial_N \omega_{M_1 \ldots M_n}. \tag{2.29}$$

In terms of tangent space indices, these relations become

$$\begin{aligned}
\omega &= \frac{1}{n!} E^{A_n} \wedge \cdots \wedge E^{A_1} \omega_{A_1 \ldots A_n} \\
d\omega &= \frac{1}{n!} E^{A_n} \wedge \cdots \wedge E^{A_1} \wedge E^B \left(D_B \omega_{A_1 \ldots A_n} + \frac{n}{2} T_{BA_1}{}^C \omega_{CA_2 \ldots A_n} \right),
\end{aligned} \tag{2.30}$$

where the torsion two-form T is defined by

$$T^A = dE^A = \frac{1}{2} E^C \wedge E^B T_{BC}{}^A. \tag{2.31}$$

Explicitly, we get that the only non-zero component of the torsion is

$$T_{\beta\gamma}^{bc[\alpha_1\alpha_2]} = -2i\delta_\beta{}^{[\alpha_1}\delta_\gamma{}^{\alpha_2]}\Omega^{bc}. \tag{2.32}$$

We will also need to take pull-backs of superforms to bosonic manifolds embedded in superspace. Let D be such a manifold, parametrized by σ^i, $i = 1, \ldots, \dim(D)$. The pull-back of the differentials in tangent space is

$$E^A = d\sigma^i \partial_i Z^M E_M{}^A \equiv d\sigma^i E_i^A, \tag{2.33}$$

where $Z^M = Z^M(\sigma)$ is the embedding function for the manifold D in target superspace and ∂_i is the derivative with respect to σ^i. This means that the pull-backs are

$$\begin{aligned}
E^{[\alpha\beta]} &= d\sigma^i \left(\partial_i X^{\alpha\beta} + i\Omega^{ab} \Theta_a^{[\alpha} \partial_i \Theta_b^{\beta]} \right) \\
E_a^\alpha &= d\sigma^i \partial_i \Theta_a^\alpha.
\end{aligned} \tag{2.34}$$

3. The interaction terms

Our goal in this section is to construct supersymmetric interaction terms that couple a string to a tensor multiplet background. This background is assumed to be described by the superfield Φ^{ab} in section 2.2. In particular, it obeys the constraints (2.6)–(2.7).

The string world-sheet Σ is embedded in target superspace. Parametrizing Σ by a set of coordinates σ^i, $i = 1, 2$, the embedding may be described by a function $Z^M(\sigma)$. We denote the components of $Z^A = Z^M E_M{}^A$ by $X^{\alpha\beta}$ and Θ_a^α. Under a supersymmetry transformation with parameter η, the embedding functions transform according to

$$\delta X^{\alpha\beta} = i\Omega^{ab} \eta_a^{[\alpha} \Theta_b^{\beta]} \tag{3.1}$$

$$\delta \Theta_a^\alpha = -\eta_a^\alpha. \tag{3.2}$$

Note that this means that both $E^{\alpha\beta}$ and E_a^α are invariant under supersymmetry.

The first coupling term is a standard Nambu-Goto term, with the string tension replaced by an expression in the superfield Φ^{ab}, evaluated at the locus of the string. Explicitly, we have that

$$S_{\text{NG}} = -\int_{\Sigma} d^2\sigma \sqrt{\Phi\cdot\Phi}\sqrt{-G}\,, \tag{3.3}$$

where G denotes the determinant of the induced world-sheet metric G_{ij}, given by

$$G_{ij} = \frac{1}{4}\epsilon_{\alpha\beta\gamma\delta}E_i^{\alpha\beta}E_j^{\gamma\delta}\,, \tag{3.4}$$

and the scalar product should be read as

$$\Phi\cdot\Phi = \frac{1}{4}\Omega_{ac}\Omega_{bd}\Phi^{ab}\Phi^{cd}\,. \tag{3.5}$$

Under a supersymmetry transformation, $\sqrt{-G}$ is obviously invariant, while $\sqrt{\Phi\cdot\Phi}$ (being a superfield evaluated at Z) transforms according to

$$\delta\sqrt{\Phi\cdot\Phi} = \left(\eta_a^\alpha Q_\alpha^a + \delta X^{\alpha\beta}\partial_{\alpha\beta} + \delta\Theta_a^\alpha \partial_\alpha^a\right)\sqrt{\Phi\cdot\Phi} = 0\,, \tag{3.6}$$

where we have used eqs. (2.2), (3.1) and (3.2). Note that this is true only if Φ obeys the differential constraint (2.7). Thus, S_{NG} is invariant under supersymmetry, modulo terms proportional to the equations of motion for the free tensor multiplet.

The second coupling term is of Wess-Zumino type and is written as

$$S_{\text{WZ}} = \int_D F\,, \tag{3.7}$$

where the integration is over a three-manifold D such that $\partial D = \Sigma$. F is a super-three-form with components

$$\begin{aligned}
F_{[\alpha_1\alpha_2][\beta_1\beta_2][\gamma_1\gamma_2]} &= \frac{1}{6}\left(H_{\alpha_1\beta_1}\epsilon_{\alpha_2\beta_2\gamma_1\gamma_2} + H_{\beta_1\gamma_1}\epsilon_{\beta_2\gamma_2\alpha_1\alpha_2} + H_{\gamma_1\alpha_1}\epsilon_{\partial_2\alpha_2\beta_1\beta_2}\right)\\
F^a_{\alpha[\beta_1\beta_2][\gamma_1\gamma_2]} &= \frac{i}{4}\left(\Psi^a_{\beta_1}\epsilon_{\beta_2\alpha\gamma_1\gamma_2} - \Psi^a_{\gamma_1}\epsilon_{\gamma_2\alpha\beta_1\beta_2}\right)\\
F^{ab}_{\alpha\beta[\gamma_1\gamma_2]} &= \frac{i}{2}\Phi^{ab}\epsilon_{\alpha\beta\gamma_1\gamma_2}\\
F^{abc}_{\alpha\beta\gamma} &= 0\,,
\end{aligned} \tag{3.8}$$

where antisymmetrization in α_1,α_2 etc. is understood. Using eq. (2.30), it can be shown that

$$\begin{aligned}
dF = {} & E^{\delta_1\delta_2}\wedge E^{\gamma_1\gamma_2}\wedge E^{\beta_1\beta_2}\wedge E^{\alpha_1\alpha_2}\left[\frac{1}{8}\partial_{[\alpha_1\alpha_2}H_{\beta_1]\gamma_1}\epsilon_{\beta_2\gamma_2\delta_1\delta_2}\right] + \\
& + E^{\delta_1\delta_2}\wedge E^{\gamma_1\gamma_2}\wedge E^{\beta_1\beta_2}\wedge E_a^\alpha\left[\frac{i}{4}\epsilon_{\beta_2\alpha\delta_1\delta_2}\partial_{[\gamma_1\gamma_2}\Psi^a_{\beta_1]} + \frac{1}{12}\epsilon_{\beta_2\gamma_2\delta_1\delta_2}\times\right.\\
& \qquad\qquad \left.\times\left(D^a_\alpha H_{\beta_1\gamma_1} - i(\partial_{\alpha\beta_1}\Psi^a_{\gamma_1} + \partial_{\alpha\gamma_1}\Psi^a_{\beta_1})\right)\right] + \\
& + E^{\delta_1\delta_2}\wedge E^{\gamma_1\gamma_2}\wedge E_b^\beta\wedge E_a^\alpha\left[\frac{i}{4}\epsilon_{\gamma_2\beta\delta_1\delta_2}\left(D^a_\alpha\Psi^b_{\gamma_1} + \Omega^{ab}H_{\alpha\gamma_1} - 2\partial_{\alpha\gamma_1}\Phi^{ab}\right)\right] + \\
& + E^{\delta_1\delta_2}\wedge E_c^\gamma\wedge E_b^\beta\wedge E_a^\alpha\left[\frac{i}{4}\epsilon_{\beta\gamma\delta_1\delta_2}\left(D^a_\alpha\Phi^{bc} + i\left(\Omega^{ab}\Psi^c_\alpha + \Omega^{ca}\Psi^b_\alpha + \frac{1}{2}\Omega^{bc}\Psi^a_\alpha\right)\right)\right] + \\
& + E_d^\delta\wedge E_c^\gamma\wedge E_b^\beta\wedge E_a^\alpha\left[\frac{1}{4}\epsilon_{\alpha\beta\gamma\delta}\Omega^{ab}\Phi^{cd}\right].
\end{aligned} \tag{3.9}$$

From (2.6) and eqs. (2.9)–(2.14) it is apparent that F is closed (i.e. $dF = 0$) when the equations of motion for the superfields are imposed. Invariance under supersymmetry for the Wess-Zumino term is shown in the same way as for the Nambu-Goto term.

4. The local fermionic symmetry

The fermionic fields Θ defined above contain twice as many components as they should (taking supersymmetry into account). The additional components are removed by the action of a local fermionic world-sheet symmetry, called κ-symmetry.

A κ-transformation of the field $Z^M(\sigma)$ is

$$\delta_\kappa Z^M = \kappa^A E_A{}^M \,, \tag{4.1}$$

where the (non-constant) parameter κ^A only contains a fermionic part κ^α_a. From this we find the following κ-variations of the embedding fields

$$\delta_\kappa X^{\alpha\beta} = i\Omega^{ab}\kappa_a^{[\alpha}\Theta_b^{\beta]} \tag{4.2}$$

$$\delta_\kappa \Theta^\alpha_a = \kappa^\alpha_a \,. \tag{4.3}$$

The corresponding transformation of the (pull-back of the) tangent space differential E^A is

$$\delta_\kappa E^A = d\kappa^A - \kappa^B E^C T_{CB}{}^A \,, \tag{4.4}$$

meaning that

$$\begin{aligned} \delta_\kappa E^{[\alpha\beta]} &= 2i\Omega^{ab}\kappa_a^{[\alpha} d\Theta_b^{\beta]} \\ \delta_\kappa E^\alpha_a &= d\kappa^\alpha_a \,. \end{aligned} \tag{4.5}$$

In these expressions, the exterior derivative d should be understood as $d = d\sigma^i\partial_i$.

The parameters κ are subject to the constraint

$$\Gamma^\alpha{}_\beta \kappa^\beta_a = \gamma_a{}^b \kappa^\alpha_b \,, \tag{4.6}$$

where

$$\begin{aligned} \Gamma^\alpha{}_\beta &= \frac{1}{2}\frac{1}{\sqrt{-G}}\epsilon^{ij}E_i^{\alpha\gamma}E_j^{\delta\epsilon}\epsilon_{\beta\gamma\delta\epsilon} \\ \gamma_a{}^b &= \frac{1}{\sqrt{\Phi\cdot\Phi}}\Omega_{ac}\Phi^{cb} \,. \end{aligned} \tag{4.7}$$

Obviously, $\Gamma^\alpha{}_\alpha = 0$ and $\gamma_a{}^a = 0$. One may also show that $\Gamma^\alpha{}_\beta\Gamma^\beta{}_\gamma = \delta^\alpha{}_\gamma$ and $\gamma_a{}^b\gamma_b{}^c = \delta_a{}^c$. This means that the condition (4.6) eliminates half of the components in κ.

The terms S_{NG} and S_{WZ} are not κ-symmetric by themselves, but a certain linear combination of them is. This is possible if the κ-variation of F is not only closed, but also exact, i.e.

$$\delta_\kappa F = d\omega_\kappa \,. \tag{4.8}$$

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By Stokes' theorem, the variation of the Wess-Zumino term becomes

$$\delta_\kappa S_{\rm WZ} = \int_\Sigma \omega_\kappa \,. \tag{4.9}$$

Explicitly, we find that

$$\begin{aligned} \delta_\kappa F &= \frac{1}{3!}\left(3d(\delta_\kappa Z^P) \wedge dZ^N \wedge dZ^M F_{MNP} + dZ^P \wedge dZ^N \wedge dZ^M \delta_\kappa Z^Q \partial_Q F_{MNP}\right) \\ &= \frac{1}{2} d\left(\delta_\kappa Z^P dZ^N \wedge dZ^M F_{MNP}\right), \end{aligned} \tag{4.10}$$

where we have used that $dF = 0$, which is valid when the tensor multiplet fields obey the free equations of motion. This means that

$$\omega_\kappa = d\sigma^i \wedge d\sigma^j \epsilon_{\alpha\beta\gamma\delta} \kappa^\alpha_a E^{\beta\gamma}_i \left(\frac{i}{4} E^{\delta\varepsilon}_j \Psi^a_\varepsilon - \frac{i}{2} \partial_j \Theta^\delta_b \Phi^{ab}\right). \tag{4.11}$$

We now turn to the variation of the Nambu-Goto term. We find, using eq. (4.6), that

$$\begin{aligned} \delta_\kappa \sqrt{\Phi \cdot \Phi} &= \frac{i}{4} \frac{1}{\sqrt{-\det G}} \epsilon^{ij} \epsilon_{\alpha\beta\gamma\delta} \kappa^\alpha_a E^{\beta\gamma}_i E^{\delta\varepsilon}_j \Psi^a_\varepsilon & (4.12) \\ \delta_\kappa \sqrt{-\det G} &= -\frac{i}{2} \frac{1}{\sqrt{\Phi \cdot \Phi}} \epsilon^{ij} \epsilon_{\alpha\beta\gamma\delta} \kappa^\alpha_a E^{\beta\gamma}_i \partial_j \Theta^\delta_b \Phi^{ab}\,. & (4.13) \end{aligned}$$

This yields immediately that

$$\delta_\kappa \left(S_{\rm NG} + S_{\rm WZ}\right) = 0\,. \tag{4.14}$$

Thus, this specific linear combination of coupling terms is both κ-symmetric and supersymmetric. This means that the interaction

$$S = -\int_\Sigma d^2\sigma \sqrt{\Phi \cdot \Phi} \sqrt{-G} + \int_D F \tag{4.15}$$

describes the supersymmetric coupling of a self-dual string to a tensor multiplet background.

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